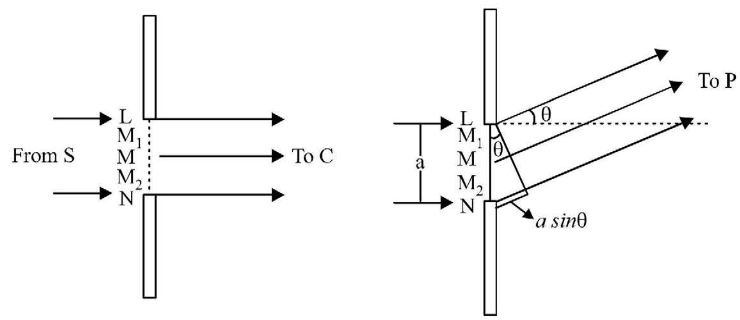
DIFFRACTION Section - 6

Definition : The bending of light around the corners of an opaque object or aperture is called diffraction.

❒ Diffraction by a Single Slit :



The figure above shows a parallel beam of light coming from a source S and falling on a slit of width a in a plane perpendicular to the direction of travel of the beam. A screen is kept at a distance D from the slit, parallel to it.

➤ Diffraction Pattern :

The beam, after passing through the slit, does not illuminate only the region on the screen directly in front of the slit, but spreads beyond as well, forming a pattern with alternating bright and dark regions.

To understand this, we use Huygens principle, which says that every point on a wavefront can be considered a secondary source. So, since one of the wavefronts of the wave is coincident with the slit, we divide it into parts and consider the path difference between the light reaching a point on the screen from each of these parts, i.e. secondary sources.

➤ Central Maxima :

Secondary waves from points equidistant from the centre of slit lying in portion LM and MN of the wavefront travel the same distance in reaching the point C.

$\therefore$ Path difference between them is zero.

Resulting in the maximum intensity at point C.

Now consider a secondary wave traveling in a direction making an angle θ. Let these waves reach at point P on the screen. Thus, intensity at point P will depend upon the path difference

➤ Secondary Minima :

Let path difference between waves from L and N reaching P be λ. Then, the whole wavefront can be divided into two equal halves LM and MN such that the path difference between the secondary waves from L and M reaching point P will be $\lambda/2$. So, for every point in the upper half, there is a corresponding point in the lower half such that phase difference between them is π. So, destructive interference takes place and we get minima.

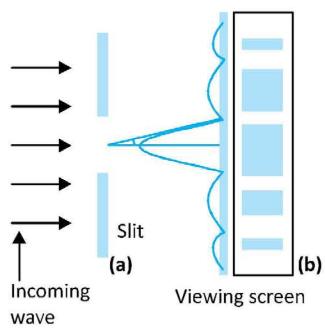
Similarly, we can divide the slit into four "quarters", each of width $\frac{a}{4}$. Then, the path difference between two points at a distance $\frac{a}{4}$, one in the top-most quarter, and the other in the second quarter from the top will be $\frac{a}{4}\sin\theta$. Therefore, at a point on the screen where this path difference becomes equal to $\frac{\lambda}{2}$, all the secondary waves reaching the screen from the top quarter destructively interfere with all the secondary waves reaching the screen from the second quarter from the top. It is also simple to see that at the same point, all the secondary waves from the third quarter destructively interfere with all the secondary waves from the lowest quarter. Therefore, this point is a point of zero brightness, a point of minima.

Therefore, the point where $\frac{a}{4}\sin\theta = \frac{\lambda}{2}$ is a point of minima.

We can similarly argue by dividing the slit into 6, 8, 10, 12... (in general $2n$, for an integer n) parts that a point where $\frac{a}{2n}\sin\theta = \frac{\lambda}{2}$ is a point of minima.

So, a point on the screen with angular position θ is a point of minima if : $\boxed{a\sin\theta = n\lambda}$; $n \neq 0$

Secondary Maxima :



Intensity distribution and photograph of fringes due to diffraction at angle slit.

It can also be shown that a point on the screen with angular position θ is a point of maxima if :

$$\boxed{a\sin\theta = (2n-1)\frac{\lambda}{2}}$$

Width of Central Maximum :

$$\beta = \frac{2\lambda D}{a}\left[\theta = \begin{matrix}\text{Angular width}\\ \text{of central maxima}\end{matrix} = \frac{\beta}{D} = \frac{2\lambda}{a}\right] : \begin{matrix}\text{Size of central maxima increases}\\ \text{when the slit width } a \text{ decreases}\end{matrix}$$

$$\text{Also } \underset{(\text{Not Central})}{\text{Width of Maxima}} = \text{Width of Minima} = \frac{D\lambda}{a}(\text{Linear width}) \text{ and } \frac{\lambda}{a}(\text{Angular width})$$

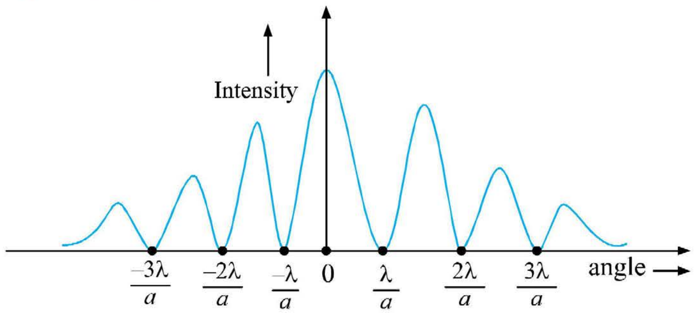
Conclusion :

(i) The intensity of subsidiary maxima decreases with distance from the central maximum. (As distance from central max increases ; smaller part of slit contributes in maxima)

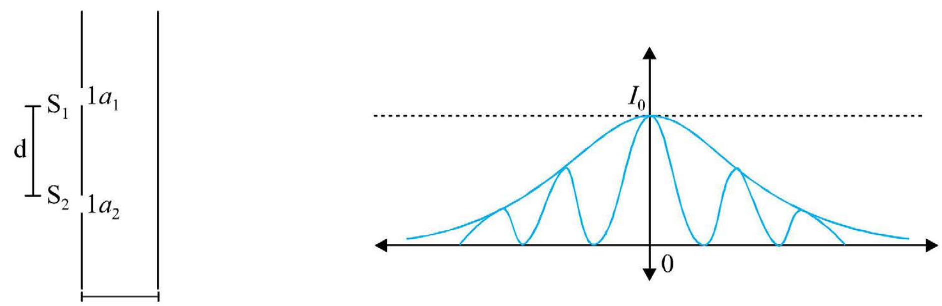
(ii) If the width of the slit is decreased, the subsidiary maxima shift away from the central maximum.

(iii) With the decrease in wavelength, the diffraction bands become narrow and crowded.

➤ Intensity-Distribution Curve :



➤ Double Slit Interference & Diffraction:



In Young's double slit experiment, the pattern observed on the screen is actually the result of the interference of the diffraction pattern of each slit. Thus, the brightness of the interference maxima goes down as we move away from the central maxima. As most of the brightness occurs in the central diffraction maxima, the number of bright fringes observed in the interference pattern is:

$$n = \frac{\text{Width of the central diffraction maxima}}{\text{Fringe width of interference pattern}} = \frac{\left(\frac{2\lambda D}{a}\right)}{\left(\frac{\lambda D}{d}\right)} = \frac{2d}{a}$$

Illustration - 11 *What will happen if we close one of the slit of YDSE ?*

SOLUTION :

Now, a single slit diffraction pattern is observed on screen. The centre of the central Bright fringe appears at a point which lies on a straight line. SS_1 or SS_2 as the case may be.

❒ The Validity of Ray optics (Fresnel Distance) :

Z_F is the distance after which the spreading of a parallel beam by an aperture (slit/hole) due to diffraction becomes comparable to size "*a*" of aperture i.e. it's a distance beyond which divergence of the beam of width '*a*' become significant.

It is the minimum distance at which the screen is to be placed so that the width of the central maxima is more than the size of aperture.

$$\Rightarrow \frac{D\lambda}{a} \geq a \quad \Rightarrow \quad D \geq \frac{a^2}{\lambda}$$

The Fresnel distance Z_F is thus given by $Z_F = \frac{a^2}{\lambda}$ (a = size of aperture)

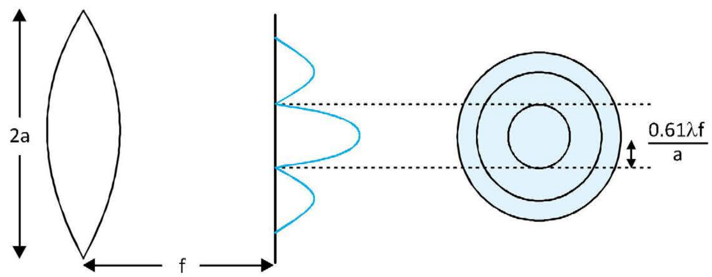
- For distances much smaller than Z_F, the spreading due to diffraction is smaller compared to the size of the beam. It becomes comparable when the distance is approximately Z_F. For distances much greater than Z_F, the spreading due to diffraction dominates over that due to ray optics.

❒ Circular Diffraction :

(i) In a convex lens; because of diffraction a parallel beam instead of getting focused at point ; gets focused to a spot of finite area.

(ii) The pattern on the focal plane would consist of a central bright region surrounded by concentric dark and bright rings where radius of the central bright region is: $\boxed{r_0 = \frac{1.22 D\lambda}{d}} \approx \frac{1.22 f\lambda}{2a}$

Where D = distance of screen from Lens (= f) ; d = aperture of the lens (= $2a$).



A parallel beam of light is incident on a convex lens.
Because of diffraction effects, the beam gets focused to a spot of radius ≈ 0.61 λf/a

Illustration - 12 *In Fraunhoffer diffraction due to a narrow slit, a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima is 5 mm on either side of the central maximum, find the wavelength of light. If your answer is $n \times 10^{-7}$ m, find the value of n.*

SOLUTION : (5)

$b \sin\theta = \lambda$

$\therefore \quad \theta = \frac{\lambda}{b}$

$\therefore \quad \lambda = b\theta = b \times \frac{5 \times 10^{-3}}{2}$

$= \frac{0.2 \times 10^{-3} \times 5 \times 10^{-3}}{2} = 5 \times 10^{-7} m$

$\therefore \quad n = 5$

Illustration - 13 *A linear aperture with width 0.2 mm is placed immediately infront of a lens of focal length 60 cm. This aperture is illuminated normally by a parallel beam whose wavelength is 500 nm. The distance between the centre and first dark band of diffraction pattern on a screen placed 60 cm from lens is $3x \times 10^{-2}$ cm. Find the value of x.*

SOLUTION : (5)

Here, $b \sin\theta = \lambda$

or $\sin\theta = \frac{\lambda}{b}$ or $\theta = \frac{\lambda}{b}$

or $\frac{Y}{f} = \frac{\lambda}{b}$

$\therefore \quad Y = \frac{\lambda}{b} f = \frac{500 \times 10^{-9} \times 60 \times 10^{-2}}{0.2 \times 10^{-3}}$

$= 15 \times 10^{-4}\ m = 15 \times 10^{-2}\ cm$

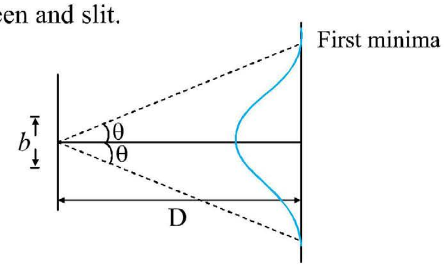
$\therefore \quad x = 5$

Illustration - 14 *Angular width of central maximum in the Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelentgth* 6000Å. *When the slit is illuminated by light of another wavelength, the angular width decreases by* 30%. *Calculate the wavelength of the light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.*

SOLUTION :

(i) Given $\lambda = 6000$ Å

Let b the width of slit and D the distance between screen and slit.



First minima is obtained at $b \sin\theta = \lambda$

or $\quad b\theta = \lambda$ as $\sin\theta = \theta$ or $\theta = \dfrac{\lambda}{b}$

Angular width of firs maxima $= 2\theta = \dfrac{2\lambda}{b} = \lambda$

Angular width will decrease by 30% when λ is decreased by 30%.

Therefore, new wavelength

$$\lambda' = \left\{(600) - \left(\frac{30}{100}\right)6000\right\} \text{ Å}$$

$$\lambda' = 4200\text{Å}$$

(ii) When the apparatus is immersed in a liquid of refractive index μ, the wavelength is decreased μ times. Therefore,

$$4200 \text{ Å} = \frac{6000 \text{ Å}}{\mu}$$

$$\mu = \frac{6000}{4200} \Rightarrow \mu = 1.429 \approx 1.43$$

POLARISATION Section - 7

Electromagnetic Waves :

Polarisation : As known, Electromagnetic Waves consist of oscillating electric and magnetic fields. The direction of the electric field vector and the magnetic field vector are mutually perpendicular as well as perpendicular to the direction of wave propagation. The direction of propagation and the direction of the electric field vector fix the direction of the magnetic field vector.

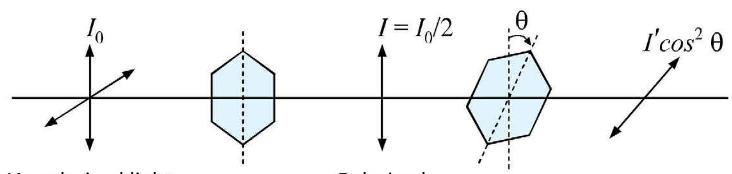
Unpolarised EM Wave : When the electric field vector is randomly oriented in the plane perpendicular to the wave propagation direction, the light is said to be unpolarised.

Linearly/Plane Polarised Light : When the electric field of the Electromagnetic Wave is confined to vary in a particular direction in the plane perpendicular to the wave direction, then the light is said to be plane polarized.

Polaroids : A polaroid consists of long chain molecules aligned in a particular direction. The electric vectors (associated with the propagating light wave) along the direction of the aligned molecules get absorbed. Thus, if an unpolarised light wave is incident on such a polaroid then the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules; this direction is known as the pass-axis of the polaroid.

It is observed that when unpolarised light from a sodium lamp, is made to pass through a randomly oriented polaroid, then the intensity of light coming out of the polaroid is half of that was incident onto the polaroid, whatever be the orientation of the polaroid. However, when another polaroid is introduced after the first one, then the intensity of the light coming out of the second Polaroid is given by $I = I' \cos^2\theta$ [$\because$ $E \cos$ è component will pass through the pass axis of 2nd Polaroid] where I' is the intensity of the light coming out from the Polaroid 1 and θ is the angle between the pass axis of the two polaroids.

This is known as the Malus' law :



Both the polaroids are kept parallel to each other.

Uses of Polaroids : Used in sunglasses, windowpanes, photographic cameras, 3D movie cameras.

Scattering of Light :

When light travels through a medium, it gets scattered in directions different from its original direction by the particles of the medium. These particles absorb the light and re-radiate it in a different, random direction. The wavelength of the light remains unaffected by scattering.

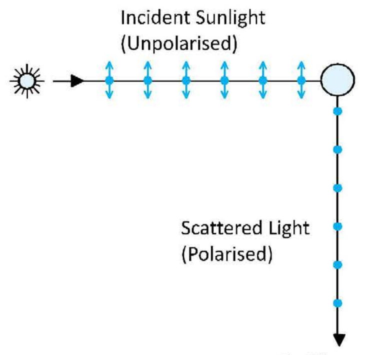
(a) When the typical size of the particles in the medium (say a) is much smaller than the wavelength (λ) of the light, i.e. $a << \lambda$, the amount of scattering is inversely proportional to λ^4, i.e. shorter wavelengths are scattered much more than longer wavelengths, i.e. blue colour is scattered much more than red. This is known as Rayleigh scattering.

An example is the scattering of light by the molecules of the gases in the earth's atmosphere. This explains the blue colour of the sky. Red colour is used in danger signs, traffic lights and aircraft warning light atop tall buildings because it suffers the least amount of scattering and continues to travel in the original direction for longer distances. This is also why the sun and the moon appear reddish when they are near the horizon, as the light coming from them has to pass through a thicker layer of the atmosphere, and this distance is enough to filter out all except the longer wavelengths, i.e. the red-orange shades.

(b) When the typical size of the particles in the medium is much larger than the wavelength of the light, i.e. $a << \lambda$, all wavelengths get scattered nearly equally. This is why clouds which contain water droplets are generally white.

Polarisation by Scattering :

Oscillating electric field of an unpolarised light can be thought to consist of two independent and perpendicular electric fields as shown by dot and arrow in the figure shown. [There is no phase relation between these two perpendicular electric fields]

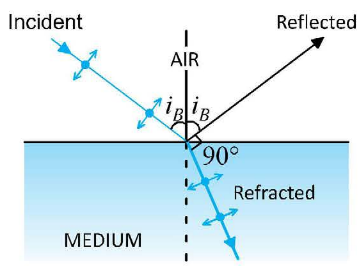


(a)

Under the influence of the electric field of the incident wave, the electrons in the molecules acquire components of motion in both these directions. When an observer looks at 90° to the direction of the sun, charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component, and only the electrons oscillating parallel to the dots radiate energy towards the observer. It is polarized perpendicular to the plane of the figure. This explains the polarisation of scattered light from the sky.

Polarisation by Reflection :

When unpolarised light is incident on the boundary between two transparent media, the reflected light is polarized with its electric vector perpendicular to the plane of incidence in the case when the refracted and reflected rays make a right angle with each other.



(b)

The angle of incidence in this case is called Brewster's angle and is denoted by i_B.

$$i_B + r = \pi/2$$

From Snell's Law :

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin(\pi/2 - i_B)} \Rightarrow \mu = \frac{\sin i_B}{\cos i_B} = \tan i_B \quad (\mu \rightarrow \text{refractive index of denser medium})$$

This is known as Brewster's Law

Note : Whenever unpolarized beam of light is incident at the Brewster's angle on an interface of two media, only the light with electric field vector perpendicular to the plane of incidence will be reflected. Now By using a good polarizer, if we remove all the light with its Electric field vector perpendicular to plane of incidence and let this light be incident on prism surface at Brewster angle, there will be no reflection and total transmission of light.

Illustration - 15 *Two polarisers A and B are placed in the path a beam of circularly polarised light of intensity I_0 such that their pass-axes are perpendicular. Now, a third polariser C is introduced between the two polarisers, such that its pass-axis makes an angle θ with the pass-axis of A. Find:*

(a) *the intensity of the emergent beam before the polariser C is introduced, I_{AB}*

(b) *the intensity of the emergent beam after the polariser C is introduced, I_{ACB}*

(c) *the value of θ for which the intensity I_{ACB} becomes maximum*

SOLUTION :

We need to remember two important things:

(a) Whenever a beam of circularly polarised light of intensity I_0 passes through a polariser, its intensity becomes $\frac{I_0}{2}$

(b) When a beam of plane polarised light of intensity passes through a polariser whose pass-axes makes an angle θ with the plane of polarisation of the incident beam (i.e. the plane in which the electric field of the incident beam oscillates), then its intensity becomes $I_0 \cos^2\theta$.

Also, the beam is now polarised along the pass-axes of the polariser.

Before C is introduced

Intensity emerging from polariser A,

$$I_A = \frac{I_0}{2}$$

Intensity emerging from polariser B,

$$I_{AB} = I_A \cos^2 90^o = 0$$

After C is introduced

Intensity emerging from polariser A,

$$I_A = \frac{I_0}{2}$$

Intensity emerging from polariser C,

$$I_{AC} = I_A \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

Intensity emerging from polariser B,

$$I_{ACB} = I_{AC}\cos^2(90° - \theta)$$

$$= \frac{I_0}{2}\cos^2\theta \sin^2\theta = \frac{I_0}{8}\sin^2 2\theta$$

We can see that I_{ACB} is maximum when $\theta = 45°$, and this maximum value is $\frac{I_0}{8}$.

Illustration - 16 *A beam of unpolarised light is incident on the interface between a medium of refractive index $n_1 = \frac{2}{\sqrt{3}}$ and a medium of refractive index n_2, producing both a reflected and a refracted beam. The reflected beam is observed through a polaroid. It is observed that when the angle of incidence of the beam is 60°, the polaroid can be rotated to a position such that the reflected beam vanishes.*

(a) *Find the refractive index n_2*

(b) *To achieve the situation when the reflected beam vanishes when viewed through the polaroid, what should be the orientation of the pass-axis of the polaroid ?*

SOLUTION :

Whenever a wave of light is incident on an interface between two mediums, there is both reflection and refraction. The reflected wave in the general situation is unpolarised (i.e. it has no single direction in which its electric field oscillates) and therefore when it is viewed through a polaroid, it does not vanish for any orientation of the polaroid's pass-axis.

But when the angle of incidence is equal to Brewster's angle, $\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$, the reflected wave is plane polarised along the direction perpendicular to the plane containing the line of propagation of the incident and reflected waves. Therefore, if the pass-axis of the polaroid is oriented parallel to this plane, the reflected wave becomes invisible.

We are given that $\theta_B = 60°$.

Therefore, $n_2 = n_1 \tan\theta_B = \left(\frac{2}{\sqrt{3}}\right)\left(\sqrt{3}\right) = 2$

IN-CHAPTER EXERCISE-B

Choose the correct alternative. Only One choice is Correct :

1. *In a single slit diffraction experiment, a slit of width 0.1 mm is illuminated by light of wavelength 500 nm, and a screen is placed at a distance 2 m from the plane of the slit. The width of the central maxima will be:*

(A) 0.5 *cm* (B) 1 *cm* (C) 2 *cm* (D) 4 *cm*

2. *If blue light is used instead of red light to observe diffraction pattern from single slit. Then, diffraction pattern :*

(A) *Will expand* (B) *Will contract*
(C) *will be more clear* (D) *Will disappear*

3. *If width of slit is b and wavelength of used light is λ. Then, the condition of diffraction is:*

(A) $b \approx \lambda$ (B) $b >> \lambda$ (C) $b << \lambda$ (D) *None of these*

4. *In a single slit diffraction experiment, a slit of width 0.2 mm is illuminated by light of wavelength 500 nm, and a screen is placed at a distance 4 m from the plane of the slit. A point on the screen at a distance 2.75 cm from the central maxima is:*

(A) *a point of maxima*
(B) *a point of minima*
(C) *the mid-point between a point of maxima and a point of minima*
(D) *None of the above*

5. *In a single slit diffraction experiment, two points on the screen with angular position* $\theta = 0.0015$ rad and $\theta = 0.0040$ *rad are both points of minima. Then, the number of points of maxima between them must be at least:*

(A) 4 (B) 5 (C) 6 (D) 7

6. *A narrow slit of width 0.02 mm is illuminated by a parallel beam of light. The diffraction pattern is observed through a telescope. It is found that to reach the first minima, the telescope has to be rotated through* $1°30'$ *from the direction of the direct ray. Calculate the wavelength used.*

(A) 400 *nm* (B) 523.3 *nm* (C) 600 *nm* (D) 700 *nm*

7. *Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal:*

(A) *That the central maximum is narrower* (B) *More number of fringes*
(C) *Less number of fringes* (D) *No diffraction pattern*

8. *A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minima of the diffraction pattern, the phase difference between the rays coming from two edges of the slit is:*

(A) Zero **(B)** $\pi/2$ **(C)** π **(D)** 2π

9. *A polariser is placed in the path of a plane-polarised beam of light of intensity I_0, such that the angle between the axis of polarisation of the beam and the pass-axis of the polariser is $30°$. The intensity of the beam after it emerges from the polariser is:*

(A) $\frac{3}{8}I_0$ **(B)** $\frac{3}{4}I_0$ **(C)** $\frac{\sqrt{3}}{4}I_0$ **(D)** $\frac{\sqrt{3}}{2}I_0$

10. *Two polarisers A and B are placed in the path of a circularly polarised beam of light of intensity I_0 such that the beam is incident on polariser A first. The angle between the pass-axes of the polarisers is θ. The beam that emerges from the polariser B:*

(A) has intensity $I_0 \cos^2\theta$ and is polarised along the pass-axis of A

(B) has intensity $I_0 \cos^2\theta$ and is polarised along the pass-axis of B

(C) has intensity $\frac{I_0 \cos^2\theta}{2}$ and is polarised along the pass-axis of A

(D) has intensity $\frac{I_0 \cos^2\theta}{2}$ and is polarised along the pass-axis of B

11. *Three polarisers A, B and C are placed in the path of a circularly polarised beam of light of intensitysuch that the beam is incident on polariser A first. Initially, the pass-axes of all three polarisers are oriented along the same direction. Now, the polariser B is slowly rotated. Then, until the pass-axes of B makes an angle $90°$ with its initial direction, the intensity of the emergent beam:*

(A) continuously decreases **(B)** continuously increases

(C) first decreases, then increases **(C)** first increases, then decreases

12. *Suppose an unpolarised light wave travels along the X-axis. The electric field at any instant is in the YZ-plane*

$$E = E_1 \sin(\omega t - kx + \phi)\hat{j} + E_2 \sin(\omega t - kx)\hat{k}$$

Some informations about ϕ in Column I and corresponding results given in Column II. Match the Column I with Column II and mark the correct option from the codes given below.

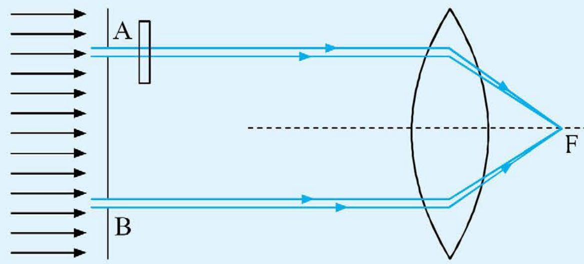
	Column I		Column II
I.	$\phi = 0$	(p)	Circularly polarised
II.	$\phi = \pi$	(q)	Elliptically polarised
III.	$\phi = \frac{\pi}{2}$ and $E_1 = E_2$	(r)	Linearly polarised
IV.	$\phi = \frac{\pi}{2}$ and $E_1 \neq E_2$	(s)	Plane polarised

Codes :

	I	II	III	IV
(A)	r, s	r, s	p	q
(B)	r, s	p	q	r
(C)	p, s	q, s	r	s
(D)	r, s	r, s	p, s	q, s

SUBJECTIVE SOLVED EXAMPLES

Example - 1 *In a modified Young's double slit experiment, a mono-chromatic uniform and parallel beam of light of wavelength 6000Å and intensity $10/\pi\ Wm^{-2}$ is incident normally on two circular apertures A and B of radii 0.001 m and 0.002m respectively. A perfect transparent film of thickness 2000Å and refractive index 1.5 for the wavelength 6000Å is placed in front of aperture A. Calculate the power received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.*



SOLUTION :

P_1 = Power transmitted through

$$A = \frac{10}{100}\left[\frac{10}{\pi}\right] Wm^{-2}\,\pi\,(0.001\,m)^2$$

P_2 power transmitted through

$$B = \frac{10}{100}\left[\frac{10}{\pi}\right] Wm^{-2} \times \pi\,(0.002)^2\,m^2 = 4\times10^{-6}W$$

$\Delta\phi =$ phase difference introduced by film.

$$= \frac{2\pi}{\lambda}(\text{path difference introduced})$$

$$= \frac{2\pi}{\lambda}(\mu - 1)\,t = \frac{\pi}{3}\ \text{radians}$$

P = power received at F

$$\Rightarrow \quad P = P_1 + P_2 + 2\sqrt{P_1 P_2}\cos\Delta\phi$$

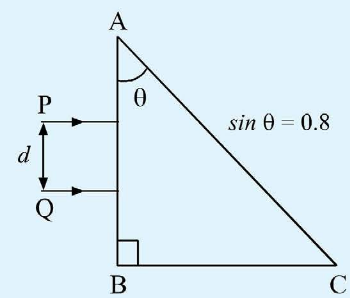
$$= 10^{-6} + 4\times10^{-6} + \sqrt{4\times10^{-12}}\cos\frac{\pi}{3}$$

$$= 7\times10^{-6}W$$

Example - 2 *Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown. The refractive index of the prism as a function of wavelength is given by the relation,*

$$\mu(\lambda) = 1.2 + (b/\lambda^2)$$

where λ is in Å and b is a positive constant. The value of b is such that the condition for total internal reflection at face AC is just satisfied for one wavelength and is not satisfied for the other.



(a) *Find the value of b*

(b) *Find the deviation of the beams transmitted through AC.*

(c) *A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and lower beams, immediately after transmission from the face AC, are 4I and I respectively, find the resultant intensity at the focus.*

SOLUTION :

For smaller wavelength λ, μ is more and hence there are more chances of total internal reflection.

$$\Rightarrow \quad \lambda = 4000\text{Å}\ \text{is just reflected at AC}$$

$$\Rightarrow \quad \mu_{4000} = \frac{1}{\sin\theta} = \frac{1}{0.8}$$

$$\Rightarrow \quad 1.2 + \frac{b}{(4000)^2} = \frac{1}{0.8} \quad \Rightarrow \quad b = 8\times10^5\ \text{Å}^2.$$

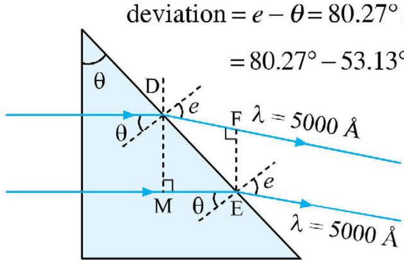
(b) $\mu_{5000} = 1.2 + \dfrac{8 \times 10^5}{(5000)^2} = 1.232$

At face AC, $\dfrac{\sin\theta}{\sin e} = \dfrac{1}{1.232}$

$\Rightarrow \quad \sin e = 0.9856 \quad \Rightarrow \quad e = 80.27°$

deviation $= e - \theta = 80.27° \sin^{-1} 0.8$

$= 80.27° - 53.13° = 27.14°.$



(c) Optical Path difference between 1 and 2 is :

$P = x_2 - x_1$

$= (ME)\,\mu - DF$

$= \mu d \tan\theta - \dfrac{d}{\cos\theta} \sin e$

$= \mu d \tan\theta - d \tan\theta \dfrac{\sin e}{\sin\theta}$

$= \mu d \tan\theta - \mu d \tan\theta = 0$

Hence the resultant intensity is :

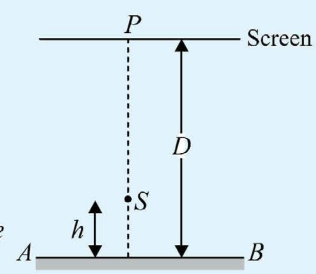
$I = I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$

Example - 3 *A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it.*

(a) *What is the shape of the interference fringes on the screen?*

(b) *Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P.*

(c) *If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the source S should be shifted so that the intensity at P again becomes maximum.*



SOLUTION :

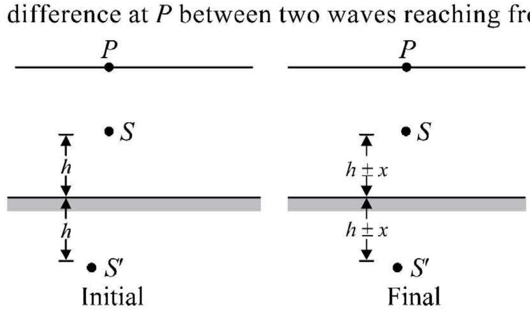
(a) Shape of the interference fringes will be circular.

(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36 I_0$.

$\therefore \quad \dfrac{I_1}{I_2} = \dfrac{I_0}{0.36 I_0} = \dfrac{1}{0.36}$ or $\sqrt{\dfrac{I_1}{I_2}} = \dfrac{1}{0.6}$

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{(\sqrt{I_1} - \sqrt{I_2})^2}{(\sqrt{I_1} + \sqrt{I_2})^2} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Intially path difference at P between two waves reaching from S and S' is $2h$.



As there is a phase differecne of π due to reflection from mirror. Therefore, for maximum intensity at P :

$$2h = \left(n - \frac{1}{2}\right)\lambda \quad \text{. . . (i)}$$

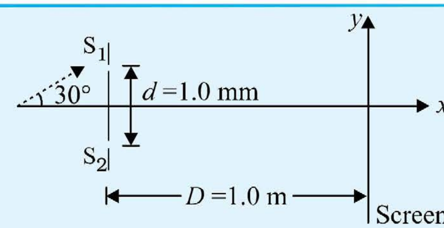
Now, let the source S is shifted by x. The path difference will be $2h + 2x$ or $2h - 2x$. For maximum intensity at P again:

$$2h + 2x = \left[n + 1 - \frac{1}{2}\right]\lambda \quad \text{. . . .(ii)} \qquad \text{or} \qquad 2h - 2x = \left[n - 1 - \frac{1}{2}\right]\lambda \quad \text{. . . . (iii)}$$

Solving Eqs. (i) and (ii) or Eqs. (i) and (iii), we get :

$$x = \frac{\lambda}{2} = \frac{600}{2} = 300\ nm$$

Example - 4 *A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure.*



(a) *If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interfrence minima on the screen.*

(b) *If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in figure), find the y-coordinates of the first minima on either side of the central maximum.*

SOLUTION :

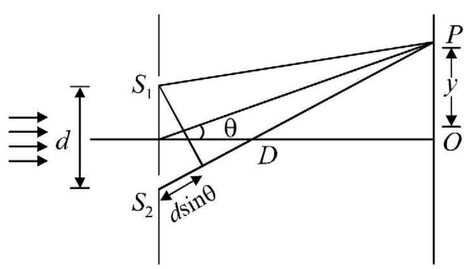
Given, $\lambda = 0.5\ mm$, $d = 1.0\ mm$, $D = 1\ m$

(a) When the incident beam falls normally :

Path difference between the two rays S_2P and S_1P is
$\Delta x = S_2 - S_1P = d \sin\theta$

For minimum intensity, $d\sin\theta = (2n-1)\dfrac{\lambda}{2}$, where n = 1, 2, 3 .....

or $\quad \sin\theta = \dfrac{(2n-1)\lambda}{2d} = \dfrac{(2n-1)0.5}{2\times 1.0} = \dfrac{2n-1}{4}$



As $\sin\theta \le 1$ therefore $\dfrac{(2n-1)}{4} \le 1$ or $n \le 2.5$

So, n can be either 1 or 2.

When $n = 1$, $\sin\theta_1 = \dfrac{1}{4}$ or $\tan\theta_1 = \dfrac{1}{\sqrt{15}}$

When $n = 2$, $\sin\theta_2 = \dfrac{3}{4}$ or $\tan\theta_2 = \dfrac{3}{\sqrt{7}}$

$\therefore \quad y = D\tan\theta = \tan\theta \quad (D = 1\ m)$

So, the position of minima will be

$$y_1 = \tan\theta_1 = \frac{1}{\sqrt{15}}\ m = 0.26\ m$$

$$y_2 = \tan\theta_2 = \frac{3}{\sqrt{7}}\ m = 1.13\ m$$

And as minima can be on either side of centre O.

Therefore there will be four minimas at positions $\pm 0.26\ m$ and $\pm 1.13\ m$ on the screen.

(b) When $\alpha = 30°$, path difference between the rays before reaching S_1 and S_2 is

$$\Delta x_1 = d\sin\alpha = (1.0)\sin 30° = 0.5\ mm = \lambda$$

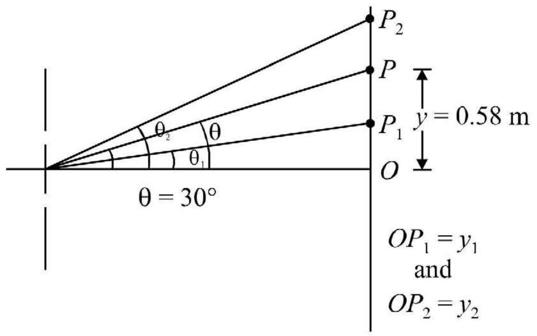
So, there is already a path difference of λ between the rays.

Position of central maximum Central maximum is defined as a point where net path difference is zero.

So, $\Delta x_1 = \Delta x_2$

or $d \sin\alpha = d\ \sin\theta \Rightarrow \theta = \alpha = 30°$

or $\tan\theta = \frac{1}{\sqrt{3}} = \frac{y_0}{D} \Rightarrow y_0 = 0.58\ m \quad (D = 1m)$



At point P, $\Delta x_1 = \Delta x_2$, Above point P $\Delta x_2 > \Delta x_1$ and Below point P $\Delta x_1 > \Delta x_2$

Now, let P_1 and P_2 be the minimas on either side of cental maxima. Then, for P_2

$$\Delta x_2 - \Delta x_1 = \frac{\lambda}{2}$$

$$\Rightarrow \Delta x_2 = \Delta x_1 + \frac{\lambda}{2} = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2}$$

$$\Rightarrow d\sin\theta_2 = \frac{3\lambda}{2}$$

$$\text{or} \quad \sin\theta_2 = \frac{3\lambda}{2d}\frac{(3)(0.5)}{(2)(1.0)} = \frac{3}{4}$$

$$\therefore \tan\theta_2 = \frac{3}{\sqrt{7}} = \frac{y_2}{D} \Rightarrow y_2 = \frac{3}{\sqrt{7}} = 1.13m$$

Similarly for P_1

$$\Delta x_1 - \Delta x_2 = \frac{\lambda}{2}$$

$$\Rightarrow \Delta x_2 = \Delta x_1 - \frac{\lambda}{2} = \lambda - \frac{\lambda}{2} = \frac{\lambda}{2}$$

$$\Rightarrow d\sin\theta_1 = \frac{\lambda}{2}$$

$$\text{or} \quad \sin\theta_1 = \frac{\lambda}{2d} = \frac{(0.5)}{(2)(1.0)} = \frac{1}{4}$$

$$\therefore \tan\theta_1 = \frac{1}{\sqrt{15}} = \frac{y_1}{D}$$

$$\Rightarrow y_1 = \frac{1}{\sqrt{15}} m = 0.26\ m$$

Therefore, y-coordinates of the first minima on either side of the central maximum are $y_1 = 0.26$ and $y_2 = 1.13\ m$.

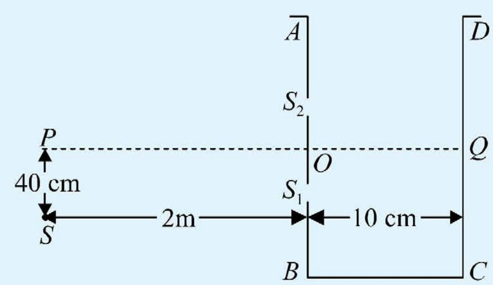
Note : In this problem $\sin\theta \approx \tan\theta \approx \theta$ is not valid as θ is large.

Example - 5 *A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passsing through O, the middle point of S_1 and S_2.*

A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside.

(a) *Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled upto OQ.*

(b) *If the central bright fringe is found to be at Q, calculate the refractive index of the liquid.*



SOLUTION :

(a) Given, $y_1 = 40\ cm$, $D_1 = 2m = 200\ cm$, $D_2 = 10\ cm$

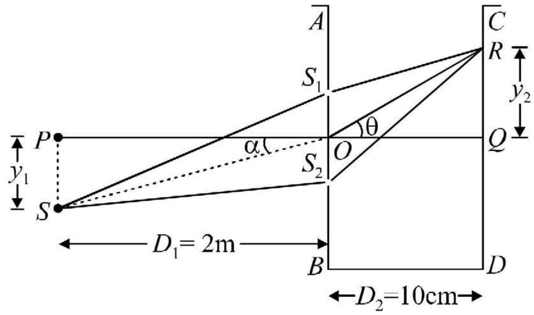
ΔX at R will be zero if $\Delta X_1 = \Delta X_2$

or $d \sin \alpha = d \sin \theta$

or $\alpha = \theta$ or $\tan \alpha = \tan \theta$

$\Rightarrow \quad \frac{y_1}{D_1} = \frac{y_2}{D_2}$

or $y_2 = \frac{D_2}{D_1} \cdot y_1 = \left(\frac{10}{200}\right)(40)\ cm = 2cm$



(b) The central bright fringe will be observed at point Q, if the path difference created by the liquid slab of thickness $t = 10cm$ or $100\ mm$ is equal to ΔX_1, so that the net path differecne at Q becomes zero.

So, $(\mu - 1)t = \Delta X_1$ or $(\mu - 1)(100) = 0.1$ or $\mu = 1.0016$

MISCELLANEOUS EXERCISE

Choose the correct options for each of the following questions. Questions marked with * may have more than one correct options.

1. If the wavelength of light used in the young's double slit experiment is λ and if d is the width of the slit, what is the angular width of the central maximum ?

(A) $sin^{-1}(\lambda/d)$ **(B)** $2sin^{-1}(\lambda/d)$

(C) $sin^{-1}(2\lambda/d)$ **(D)** $sin^{-1}(\lambda/2d)$

2. White light is used to illuminate the two slits in Young's double slit experiment. The separation between the slits is d and the distance between the screen and the slit is D (>> d). At a point on the screen directly in front of one of the slits, certain wavelength are missing. The missing wavelengths are (here $m = 0, 1, 2, \ldots$ is an integer)

(A) $\lambda = \frac{d^2}{(2m+1)D}$ **(B)** $\lambda = \frac{(2m+1)d^2}{D}$

(C) $\lambda = \frac{d^2}{(m+1)D}$ **(D)** $\lambda = \frac{(m+1)d^2}{D}$

3. In an interference experiment, 20th order maximum is observed at a point on the screen when light of wavelength 480 nm is used. If this light is replaced by light of wavelength 600 nm, the order of the maximum at the same point will be :

(A) 16 **(B)** 14 **(C)** 12 **(D)** 10

4. When a ray of light goes from a denser into a rarer medium

(A) The wavelength of light is decreased

(B) The frequency of light is increased

(C) The speed of light is increased

(D) The light undergoes a phase change of π

5. In a Young's double-slit experiment, the central bright fringe can be identified

(A) as it has greater intensity than the other bright fringes

(B) as it is wider than the other bright fringes

(C) as it is narrower than the bright fringes

(D) by using white light instead of monochromatic light

6. In a Young's double-slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be :

(A) $I_0 \cos\left(\frac{x}{\beta}\right)$ **(B)** $I_0 \cos^2\left(\frac{x}{\beta}\right)$

(C) $I_0 \cos^2\left(\frac{\pi x}{\beta}\right)$ **(D)** $\left(\frac{I_0}{4}\right)\cos^2\left(\frac{\pi x}{\beta}\right)$

7. A parallel beam of fast moving electrons is incident normally on a narrow slit. If the speed of the electrons is decreased, the angular width of central maxima :
(A) Increases (B) Decreases
(C) Remains same (D) None of these

8. A polariser is placed in the path of a beam of light of intensity I_0 travelling in vacuum towards the $+X$ direction. The beam is polarized along the Z-axis. The pass-axis of the polariser makes an angle 60° with the Z-axis. After the beam emerges from the polariser, the amplitude and direction of oscillation of its magnetic field is: (μ_0 is the permeability of vacuum, c is the speed of light in vacuum)
(A) $\sqrt{\frac{I_0 \mu_0}{c}}$, 60° with the Z-axis
(B) $\sqrt{\frac{I_0 \mu_0}{2c}}$, 60° with the Z-axis
(C) $\sqrt{\frac{I_0 \mu_0}{c}}$, 30° with the Z-axis
(D) $\sqrt{\frac{I_0 \mu_0}{2c}}$, 30° with the Z-axis

9. Light with wavelength λ falls on a diffraction grating at right angles. Find the angular dispersion of the grating as a function of diffraction angle θ.
(Given: Diffraction angle is the angle between the direction of incident light beam and any diffracted beam. The angular dispersion is the amount of change of diffraction angle per unit change of the wavelength.)
(A) $\frac{\sin\theta}{\lambda}$ (B) $\frac{\cos\theta}{\lambda}$
(C) $\frac{\tan\theta}{\lambda}$ (D) $\frac{\cot\theta}{\lambda}$

10. A beam of natural light falls on a system of $N = 6$ Nicol prisms whose transmission planes are turned each through an angle $\varphi = 30°$ with respect to that of the foregoing prism. What fraction of luminous flux passes through this system ?
(A) 0.03 (B) 0.06
(C) 0.12 (D) 0.24

ANSWERS TO IN-CHAPTER EXERCISES

A	1. 8×10^{-7} m	2. 0.288 mm	3. 2 : 1	4. 7 : 5, 49 : 25	5. 6000 Å	
	6. $\mu = 1.5$; $t = 4 \times 10^{-3}$ mm		7. 8×10^{-3} mm	8. C	9. CD	10. D
B	1. C	2. B	3. A	4. C	5. B	6. B
	7. D	8. D	9. B	10. C	11. A	12. A

ANSWERS TO MISCELLANEOUS EXERCISE

1. A	2. A	3. A	4. C	5. D	6. C
7. A	8. D	9. C	10. C		